A discrete approach to the vacuum Maxwell equations and the fine structure constant

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Abstract

We recommended consequent discrete combinatorial research in mathematical physics [7]. Here we show an example how discretization of partial differential equations can be done and that quickly unexpected new findings can result from research in this up to now unexplored area. We transformed the vacuum Maxwell equations into finite-difference equations (symmetrically and simultaneously, without leapfrog), provided simple initial conditions and studied the development of the electromagnetic fields using special software [9] [10]. The development is not trivial and depends strongly on *propagators* which are new dimensionless variables which *automatically* arise during formation of the finite differences. If their squares are too small, the initial development flattens, if they are too great, it is diverging. If, however, the squares of all propagators are equal to the fine structure constant, we noticed an initially stable wavelike development and the following peculiarities:
1. The first two wave maxima have nearly the same height. Of course this can be also coincidental.
2. The following maxima are at first slightly decreasing and then, beginning with the 6th maximum, exponentially increasing.

Contents

List of Figures

1 Introduction

This paper is an updated version of [8].

We have shown [11] that only an a priori finite mathematical model can have an exact equivalent in physical reality. This means that it implies an only finite number of basic arithmetic operations on an a priori finite numerical space[1] which can be represented without using irrational numbers.

Up to now there is not much experience in this area: Important physical equations are defined on continuous (a priori infinite) sets and often written as partial differential equations. If we want to find the natural finite basis of them, first we have to replace differential calculus by finite-difference calculus. This can soon lead to difficult combinatorics, especially in case of interactions across several dimensions. But increasing performance of computers offers new possibilities. The mentioned numerical space can be represented by finite dimensional numerical lattices (finite dimensional point lattices with numbers resp. *quantities* assigned to every point) which can be handled adequately by a computer. So we developed as help software for handling numerical lattices and for studying the results of numerical algorithms on them. All lattice points are addressed by integer coordinates and the quantities assigned to the points can be complex[2]. We used the software to study the vacuum Maxwell equations.

We converted these into finite-difference equations as directly as possible because we wanted to operate on an elementary level (a priori finite). So we had to avoid analytic methods (like usage of infinite power series, trigonometric functions, complex and matrix exponential functions, Fourier transforms) which are often used to *force* an easy to survey, stable development.

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[1]Nevertheless both can increase without boundary *when* time increases without boundary (infinite potential).

[2]Both complex rational and complex floating-point numbers are supported. Subsequently by the term *lattice* always this kind of numerical lattice is meant.

2 Material and Method

2.1 Software

The in C++ written open-ended software [9] [10] is designed for editing of numerical lattices and for generation of statistics like sums over the absolute or squared quantities of subspaces, with graphical representation. Using the ASCII file format of saved data as interface or writing own code the user can check the results of own algorithms. Additionally it is possible to define algorithms without writing code. This is done by usage of a table with several entries. Every entry defines a propagation between two lattices in form of an addition from the first (the source lattice) to the second (the destination lattice). The strength of the propagation is determined by the *propagator p*: During every iteration of the algorithm all non-zero quantities of the source lattice with maximal iteration number[3] are added to shifted locations within the destination lattice after multiplication by p.

Source and destination lattices, the multidimensional relative shift resp. offset are selectable using integer numbers, the propagator p can be a complex number, if desired. The user determines how many iterations of all additions (defined by the table) are done.

One motivation for this scheme arises from the fact that it is simple and nevertheless very general. For example it can be used to study random walks like those discussed in [7]. Note that p also can be a complex probability amplitude and is not restricted to the interval $[0, 1]$.

If the number of iterations and additions per iteration is great, the lattice(s) may occupy too much memory of the computer. Therefore it is possible to activate automatic deletion of the lattice points with smallest absolute quantities if these would occupy too much memory. Here we used this after we have checked that up to 150 iterations in our application it has not affected the 12 most significant digits of the evaluated numerical data.

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[3]The results of every iteration are indexed by an increasing iteration number n. This can be used to analyze the temporal development.

2.2 The Maxwell equations

Using SI units, the Maxwell equations in linear and isotropic media are

$$\mathbf{\nabla} \cdot \mathbf{E} = \frac{\rho}{\epsilon} \, , \tag{1}$$

$$\mathbf{\nabla} \cdot \mathbf{B} = 0 \, , \tag{2}$$

$$\mu \mathbf{J} + \mu \epsilon \frac{\partial \mathbf{E}}{\partial t} = \mathbf{\nabla} \times \mathbf{B} \, , \tag{3}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\mathbf{\nabla} \times \mathbf{E} \, , \tag{4}$$

where $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field, ρ is the charge density, $\mathbf{J}$ is the vector current density, ϵ is the permittivity and μ is the permeability. In the vacuum which is free of current and electric charge ($\mathbf{J} = 0$ and $\rho = 0$) they reduce to the *vacuum Maxwell equations* in which besides (2) also (1) vanishes and (3), (4) become

$$\frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\mu_0 \epsilon_0} \mathbf{\nabla} \times \mathbf{B} = c^2 \, \mathbf{\nabla} \times \mathbf{B} \, ,$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\mathbf{\nabla} \times \mathbf{E} \, ,$$

where ϵ_0 is the permittivity of free space, μ_0 is the permeability of free space and c is the speed of light in the vacuum. Using the definition

$$\tilde{\mathbf{B}} := c\mathbf{B}$$

we obtain

$$\frac{\partial \mathbf{E}}{\partial t} = c \quad \mathbf{\nabla} \times \tilde{\mathbf{B}} \, , \tag{5}$$

$$\frac{\partial \tilde{\mathbf{B}}}{\partial t} = -c \quad \mathbf{\nabla} \times \mathbf{E} \, . \tag{6}$$

The proportionality factor c still has the dimension ms^{-1}. We will see (cf. 2.4) that during discretization automatically the question about the natural dimensionless relationship arises.

2.3 Discretization

Let x, y, z denote Cartesian coordinates and let E_x, E_y, E_z denote the components of $\mathbf{E} = \mathbf{E}(t, x, y, z)$ and $\tilde{B}_x, \tilde{B}_y, \tilde{B}_z$ those of $\tilde{\mathbf{B}} = \tilde{\mathbf{B}}(t, x, y, z)$. With

this (5) and (6) are equivalent to

$$\frac{\partial E_x}{\partial t} = c\left(\frac{\partial \tilde{B}_z}{\partial y} - \frac{\partial \tilde{B}_y}{\partial z}\right), \ \frac{\partial E_y}{\partial t} = c\left(\frac{\partial \tilde{B}_x}{\partial z} - \frac{\partial \tilde{B}_z}{\partial x}\right), \ \frac{\partial E_z}{\partial t} = c\left(\frac{\partial \tilde{B}_y}{\partial x} - \frac{\partial \tilde{B}_x}{\partial y}\right), \quad (7)$$

$$\frac{\partial \tilde{B}_x}{\partial t} = -c\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right), \ \frac{\partial \tilde{B}_y}{\partial t} = -c\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right), \ \frac{\partial \tilde{B}_z}{\partial t} = -c\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right). \quad (8)$$

A careful consideration, however, must take into account that the differences in time and location cannot be arbitrarily small.

2.3.1 (Half) minimal differences as units

We know that there are non-vanishing minimal absolute differences dt in time and ds in location[4] so that absolute differences of t below dt and of x, y, z below ds are not existing in physical reality. Therefore it is appropriate to base the scaling on dt and ds. We will use dt as unit of time and (because we want to form minimal *symmetric* differences in location) $ds/2$ as unit of location.

2.3.2 Integer representation

Define

$$\hat{t} := \frac{t}{dt}, \ \hat{x} := \frac{2x}{ds}, \ \hat{y} := \frac{2y}{ds}, \ \hat{z} := \frac{2z}{ds} \quad \text{and} \quad (9)$$

$$\hat{\mathbf{E}}(\hat{t}, \hat{x}, \hat{y}, \hat{z}) := \mathbf{E}(t, x, y, z), \ \hat{\mathbf{B}}(\hat{t}, \hat{x}, \hat{y}, \hat{z}) := \tilde{\mathbf{B}}(t, x, y, z) \ . \quad (10)$$

The coordinates $\hat{t}, \hat{x}, \hat{y}, \hat{z}$ are all dimensionless having only integer differences. Now it is possible to form minimal symmetric finite differences using only integer coordinates. For $v \in \{x, y, z\}$ let $\hat{E}_v$, $\hat{B}_v$, E_v and $\tilde{B}_v$ denote the components of $\hat{\mathbf{E}}$, $\hat{\mathbf{B}}$, $\mathbf{E}$ and $\tilde{\mathbf{B}}$ respectively. Define

$$\hat{E}_{vt} := \hat{E}_v(\hat{t}+1, \hat{x}, \hat{y}, \hat{z}) - \hat{E}_v(\hat{t}, \hat{x}, \hat{y}, \hat{z}) = E_v(t+dt, x, y, z) - E_v(t, x, y, z) \ ,$$
$$\hat{E}_{vx} := \hat{E}_v(\hat{t}, \hat{x}+1, \hat{y}, \hat{z}) - \hat{E}_v(\hat{t}, \hat{x}-1, \hat{y}, \hat{z}) = E_v(t, x+ds/2, y, z) - E_v(t, x-ds/2, y, z) \ ,$$
$$\hat{E}_{vy} := \hat{E}_v(\hat{t}, \hat{x}, \hat{y}+1, \hat{z}) - \hat{E}_v(\hat{t}, \hat{x}, \hat{y}-1, \hat{z}) = E_v(t, x, y+ds/2, z) - E_v(t, x, y-ds/2, z) \ ,$$
$$\hat{E}_{vz} := \hat{E}_v(\hat{t}, \hat{x}, \hat{y}, \hat{z}+1) - \hat{E}_v(\hat{t}, \hat{x}, \hat{y}, \hat{z}-1) = E_v(t, x, y, z+ds/2) - E_v(t, x, y, z-ds/2) \ ,$$

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[4]They depend on further variables like energy per photon which influence both dt and ds (but not necessarily in all those cases the quotient dt/ds).

$$\hat{B}_{vt} \quad := \quad \hat{B}_v(\hat{t}+1, \hat{x}, \hat{y}, \hat{z}) - \hat{B}_v(\hat{t}, \hat{x}, \hat{y}, \hat{z}) = \tilde{B}_v(t+dt, x, y, z) - \tilde{B}_v(t, x, y, z) \; ,$$
$$\hat{B}_{vx} \quad := \quad \hat{B}_v(\hat{t}, \hat{x}+1, \hat{y}, \hat{z}) - \hat{B}_v(\hat{t}, \hat{x}-1, \hat{y}, \hat{z}) = \tilde{B}_v(t, x+ds/2, y, z) - \tilde{B}_v(t, x-ds/2, y, z) \; ,$$
$$\hat{B}_{vy} \quad := \quad \hat{B}_v(\hat{t}, \hat{x}, \hat{y}+1, \hat{z}) - \hat{B}_v(\hat{t}, \hat{x}, \hat{y}-1, \hat{z}) = \tilde{B}_v(t, x, y+ds/2, z) - \tilde{B}_v(t, x, y-ds/2, z) \; ,$$
$$\hat{B}_{vz} \quad := \quad \hat{B}_v(\hat{t}, \hat{x}, \hat{y}, \hat{z}+1) - \hat{B}_v(\hat{t}, \hat{x}, \hat{y}, \hat{z}-1) = \tilde{B}_v(t, x, y, z+ds/2) - \tilde{B}_v(t, x, y, z-ds/2) \; ,$$

and transformation of (7) and (8) into finite-difference equations yields

$$\frac{\hat{E}_{xt}}{dt} = c\left(\frac{\hat{B}_{zy}}{ds} - \frac{\hat{B}_{yz}}{ds}\right), \; \frac{\hat{E}_{yt}}{dt} = c\left(\frac{\hat{B}_{xz}}{ds} - \frac{\hat{B}_{zx}}{ds}\right), \; \frac{\hat{E}_{zt}}{dt} = c\left(\frac{\hat{B}_{yx}}{ds} - \frac{\hat{B}_{xy}}{ds}\right), \quad (11)$$

$$\frac{\hat{B}_{xt}}{dt} = -c\left(\frac{\hat{E}_{zy}}{ds} - \frac{\hat{E}_{yz}}{ds}\right), \; \frac{\hat{B}_{yt}}{dt} = -c\left(\frac{\hat{E}_{xz}}{ds} - \frac{\hat{E}_{zx}}{ds}\right), \; \frac{\hat{B}_{zt}}{dt} = -c\left(\frac{\hat{E}_{yx}}{ds} - \frac{\hat{E}_{xy}}{ds}\right). \quad (12)$$

This would be equivalent to (7) and (8) in the borderline case $dt \to 0$ and $ds \to 0$. But due to 2.3.1 the equations (11) and (12) are closer to reality. Using

$$\hat{p} := c\,\frac{dt}{ds} \qquad (13)$$

as absolute propagator they become more compact:

$$\hat{E}_{xt} \quad = \quad \hat{p}\left(\hat{B}_{zy} - \hat{B}_{yz}\right), \; \hat{E}_{yt} = \hat{p}\left(\hat{B}_{xz} - \hat{B}_{zx}\right), \; \hat{E}_{zt} = \hat{p}\left(\hat{B}_{yx} - \hat{B}_{xy}\right), \quad (14)$$

$$\hat{B}_{xt} \quad = \quad -\hat{p}\left(\hat{E}_{zy} - \hat{E}_{yz}\right), \; \hat{B}_{yt} = -\hat{p}\left(\hat{E}_{xz} - \hat{E}_{zx}\right), \; \hat{B}_{zt} = -\hat{p}\left(\hat{E}_{yx} - \hat{E}_{xy}\right). \quad (15)$$

2.4 The algorithm

Written out explicitly from (14) and (15) follows

$$
\begin{aligned}
\hat{E}_x(\hat{t}+1,\hat{x},\hat{y},\hat{z}) = \hat{E}_x(\hat{t},\hat{x},\hat{y},\hat{z}) \quad &+\quad \hat{p}\left(\hat{B}_z(\hat{t},\hat{x},\hat{y}+1,\hat{z})-\hat{B}_z(\hat{t},\hat{x},\hat{y}-1,\hat{z})\right) \\
&-\quad \hat{p}\left(\hat{B}_y(\hat{t},\hat{x},\hat{y},\hat{z}+1)-\hat{B}_y(\hat{t},\hat{x},\hat{y},\hat{z}-1)\right)\ , \\
\hat{E}_y(\hat{t}+1,\hat{x},\hat{y},\hat{z}) = \hat{E}_y(\hat{t},\hat{x},\hat{y},\hat{z}) \quad &+\quad \hat{p}\left(\hat{B}_x(\hat{t},\hat{x},\hat{y},\hat{z}+1)-\hat{B}_x(\hat{t},\hat{x},\hat{y},\hat{z}-1)\right) \\
&-\quad \hat{p}\left(\hat{B}_z(\hat{t},\hat{x}+1,\hat{y},\hat{z})-\hat{B}_z(\hat{t},\hat{x}-1,\hat{y},\hat{z})\right)\ , \\
\hat{E}_z(\hat{t}+1,\hat{x},\hat{y},\hat{z}) = \hat{E}_z(\hat{t},\hat{x},\hat{y},\hat{z}) \quad &+\quad \hat{p}\left(\hat{B}_y(\hat{t},\hat{x}+1,\hat{y},\hat{z})-\hat{B}_y(\hat{t},\hat{x}-1,\hat{y},\hat{z})\right) \\
&-\quad \hat{p}\left(\hat{B}_x(\hat{t},\hat{x},\hat{y}+1,\hat{z})-\hat{B}_x(\hat{t},\hat{x},\hat{y}-1,\hat{z})\right)\ , \\
&\phantom{-\quad \hat{p}} \tag{16} \\
\hat{B}_x(\hat{t}+1,\hat{x},\hat{y},\hat{z}) = \hat{B}_x(\hat{t},\hat{x},\hat{y},\hat{z}) \quad &-\quad \hat{p}\left(\hat{E}_z(\hat{t},\hat{x},\hat{y}+1,\hat{z})-\hat{E}_z(\hat{t},\hat{x},\hat{y}-1,\hat{z})\right) \\
&+\quad \hat{p}\left(\hat{E}_y(\hat{t},\hat{x},\hat{y},\hat{z}+1)-\hat{E}_y(\hat{t},\hat{x},\hat{y},\hat{z}-1)\right)\ , \\
\hat{B}_y(\hat{t}+1,\hat{x},\hat{y},\hat{z}) = \hat{B}_y(\hat{t},\hat{x},\hat{y},\hat{z}) \quad &-\quad \hat{p}\left(\hat{E}_x(\hat{t},\hat{x},\hat{y},\hat{z}+1)-\hat{E}_x(\hat{t},\hat{x},\hat{y},\hat{z}-1)\right) \\
&+\quad \hat{p}\left(\hat{E}_z(\hat{t},\hat{x}+1,\hat{y},\hat{z})-\hat{E}_z(\hat{t},\hat{x}-1,\hat{y},\hat{z})\right)\ , \\
\hat{B}_z(\hat{t}+1,\hat{x},\hat{y},\hat{z}) = \hat{B}_z(\hat{t},\hat{x},\hat{y},\hat{z}) \quad &-\quad \hat{p}\left(\hat{E}_y(\hat{t},\hat{x}+1,\hat{y},\hat{z})-\hat{E}_y(\hat{t},\hat{x}-1,\hat{y},\hat{z})\right) \\
&+\quad \hat{p}\left(\hat{E}_x(\hat{t},\hat{x},\hat{y}+1,\hat{z})-\hat{E}_x(\hat{t},\hat{x},\hat{y}-1,\hat{z})\right)\ .
\end{aligned}
$$

For stepwise calculation of the temporal development we can, as shown in [9], directly implement these equations in our software using 6 four-dimensional numerical lattices, 3 representing $\hat{\mathbf{E}}$, 3 representing $\hat{\mathbf{B}}$, and $\pm\hat{p}$ as propagator p (cf. 2.1). At this $\hat{p}$ is a free variable, so we have to search for reasonable values of it (i.e. automatically we have to search for the natural dimensionless relationship, as mentioned in 2.2). The time steps are uniform because both laws (5) and (6) are considered to be valid at every time, i.e. $\hat{\mathbf{E}}$ and $\hat{\mathbf{B}}$ are calculated simultaneously in (16), not alternating. Therefore this algorithm differs from leapfrog FDTD schemes [12].

The above equations (16) show that a linear superposition of the initial conditions leads to a linear superposition of the results. Therefore we chose for the purpose of clarity as simple as possible nontrivial (non-zero) initial conditions. As initial time we defined $\hat{t} = 0$ and set all initial quantities (of $\hat{E}_x, \hat{E}_y, \hat{E}_z, \hat{B}_x, \hat{B}_y, \hat{B}_z$) to zero on all locations except $\hat{E}_x$ in the origin

$(0, 0, 0, 0)$ which we set to one: $\hat{E}_x(0, 0, 0, 0) := 1$. Then we analyzed the temporal development of $\hat{E}_x$ resulting from iterative calculation of (16) and its dependence of $\hat{p}$ which by definition (13) is a positive dimensionless number.

3   Results

As expected a wavelike development results. But it is not trivial, the heights of the wave maxima are not constant. In case of $\hat{p} > 1/8$ they are increasing from the beginning, which is not realistic. Therefore we focused our interest on smaller values of $\hat{p}$.

3.1   Preliminary note

Careful evaluation is necessary because (16) bases on a simplified model, particularly it is derived from the idealized vacuum situation. Even small errors in (16) can become relevant, if many iterations are done. Therefore the calculation of the first wave is more reliable than calculations of other waves and a detailed graphical representation of the first wave is reasonable.

3.2   Short term development: The first wave

We are searching for realistic values of $\hat{p}$. If these should cause an as stable as possible wave pattern, the wave maxima should have nearly equal height. Especially the first two maxima (the maxima of the first wave which we can calculate with the best reliability) should have similar heights. Fig. 1, 2 and 3 show the first wave for three different values of $\hat{p}$.
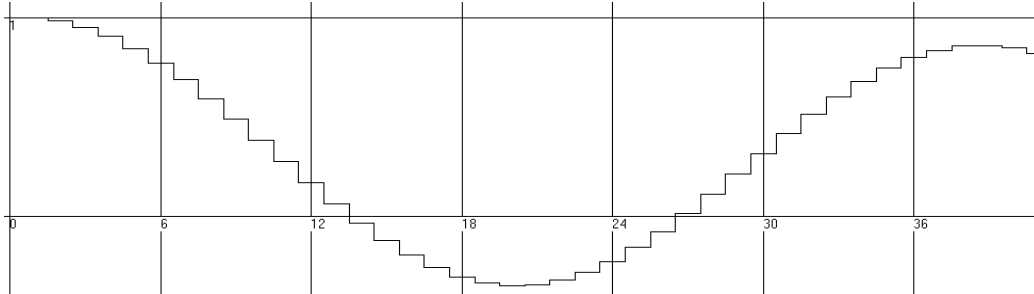
Figure 1: The first wave in case of $\hat{p} = 1/16$. Abscissa: t, Ordinate: $E_x(t, 0, 0, 0)$.
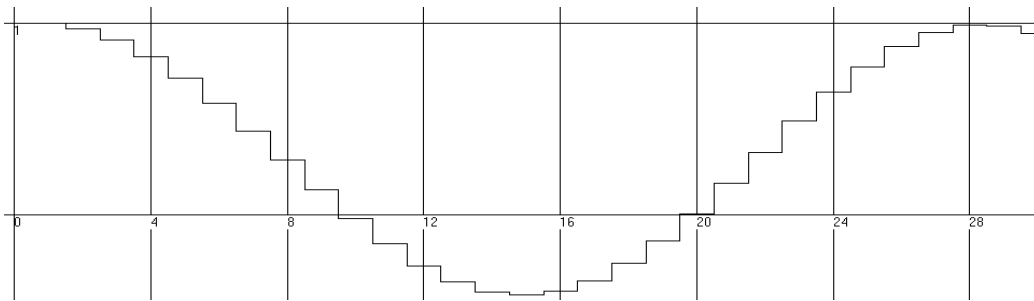

Figure 2: The first wave in case of $\hat{p} = \sqrt{\alpha} = 0.085424542921$. Abscissa: t, Ordinate: $E_x(t, 0, 0, 0)$.
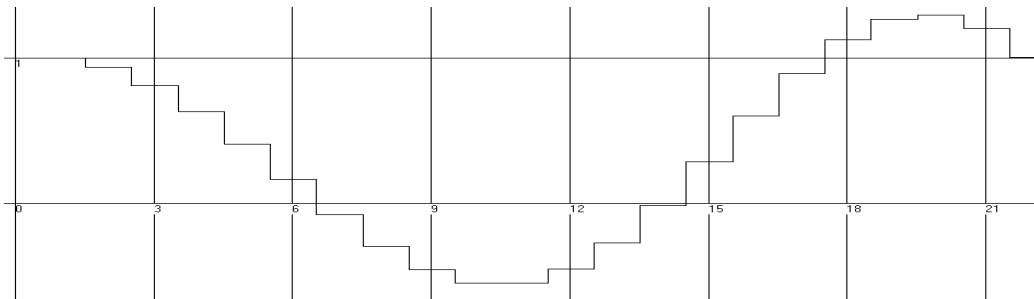

Figure 3: The first wave in case of $\hat{p} = 1/8$. Abscissa: t, Ordinate: $E_x(t, 0, 0, 0)$.

10

We see in Fig. 1 (resp. 3) that in case of $\hat{p} = 1/16$ (resp. $\hat{p} = 1/8$) the second maximum is considerably smaller (resp. greater) than the first. More interesting is Fig. 2. There

$$\hat{p} = \sqrt{\alpha} = 0.085424542921 = 1/11.7062376432$$

in which $\alpha = 1/137.03599976 = \frac{e^2}{4\pi\epsilon_0\hbar c}$ is the fine structure constant[5] as listened in [1].

The calculated quantities near the maximum at $t = 28$ are

$$\begin{aligned}
\hat{E}_x(26, 0, 0, 0) &= 0.877636902081288 , \\
\hat{E}_x(27, 0, 0, 0) &= 0.950716819197347 , \\
\hat{E}_x(28, 0, 0, 0) &= 0.98752930561647 , \\
\hat{E}_x(29, 0, 0, 0) &= 0.986271782354442 , \\
\hat{E}_x(30, 0, 0, 0) &= 0.947405056005354 .
\end{aligned}$$

So in case of $\hat{p} = \sqrt{\alpha}$ the second maximum deviates less than 2% from the first maximum which lies in the start and has the initial quantity 1. [6]

3.3 Long term development

We see in Fig. 4, 5 and 6 that $E_x(t, 0, 0, 0)$ begins to diverge considerably from the 6th maximum on. Fig. 7 and 8 show the development of $E_x(t, 0, 0, 0)$ together with $B_z(t, 0, 1, 0)$ and Fig. 9 the local distribution of E_x after 150 iterations of (16).

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[5]It can be derived very precisely from basic experiments, e.g. from measurements of the Quantum Hall Effect [13].

[6]In case of $\hat{p} = 0.08672495$ the maximum also is at $t = 28$ and differs less than 1 ppm from 1. But due to 3.1 there is no good foundation for such fine interpolation.
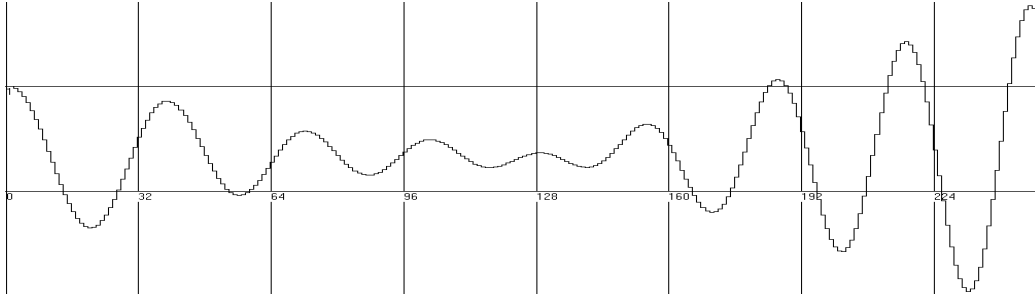
Figure 4: Long term development in case of $\hat{p} = 1/16$. Abscissa: t, Ordinate: $E_x(t, 0, 0, 0)$.
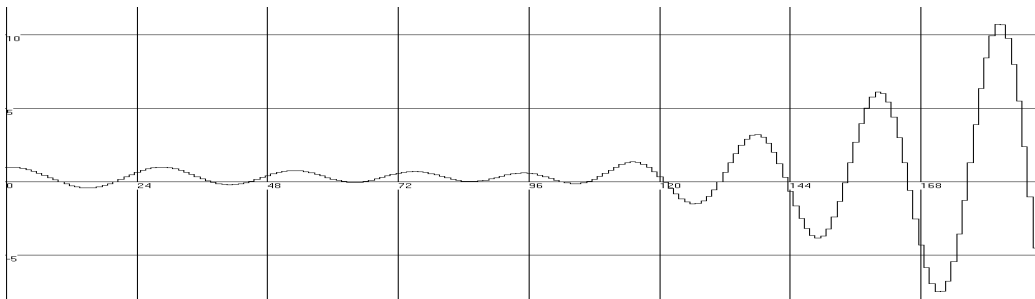

Figure 5: Long term development in case of $\hat{p} = \sqrt{\alpha}$. Abscissa: t, Ordinate: $E_x(t, 0, 0, 0)$.
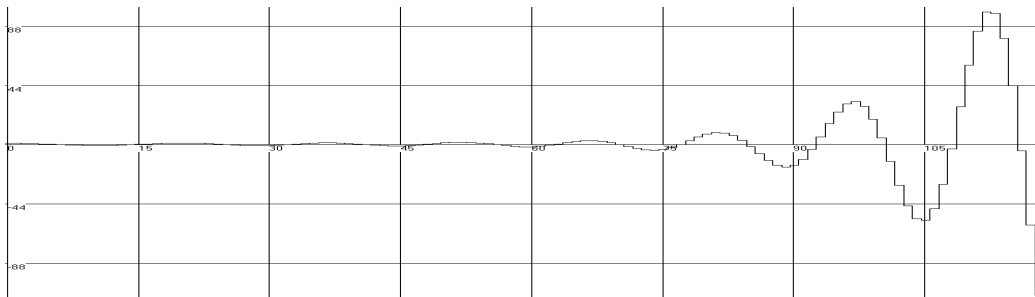

Figure 6: Long term development in case of $\hat{p} = 1/8$. Abscissa: t, Ordinate: $E_x(t, 0, 0, 0)$.
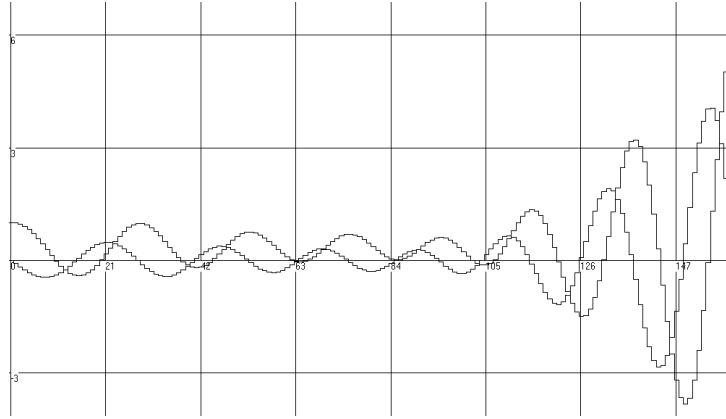
12

Figure 7: E_x together with B_z in case of $\hat{p} = \sqrt{\alpha}$. Abscissa: t, Ordinate: $E_x(t,0,0,0)$ and $B_z(t,0,1,0)$.
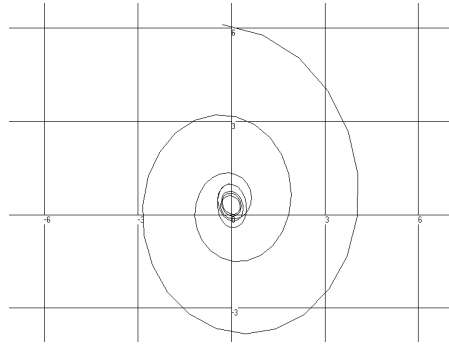

Figure 8: Relation between the fields shown in Fig. 7 ($\hat{p} = \sqrt{\alpha}$, $t \in [0,160]$). Abscissa: $B_z(t,0,1,0)$, Ordinate: $E_x(t,0,0,0)$.
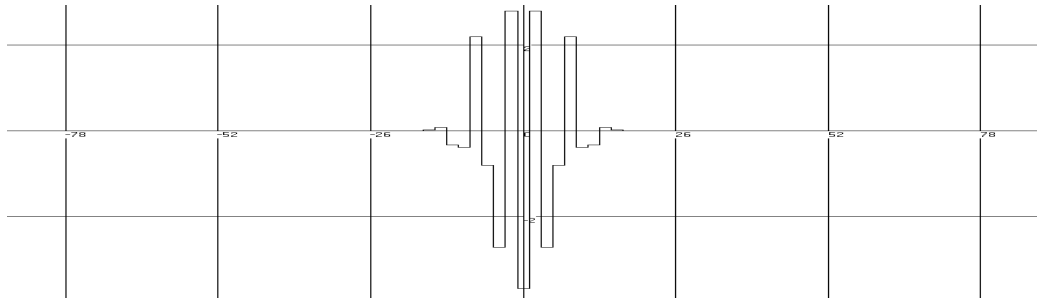

Figure 9: Local distribution of E_x after 150 iterations of (16) in case of $\hat{p} = \sqrt{\alpha}$. Abscissa: x, Ordinate: $E_x(150, x, 0, 0)$.

13

4 Comment

It was not surprising that after many iterations of (16) a wave packet results which is concentrated round the origin (Fig. 9). *As long as* there is one and the same $\hat{p}$ for all directions, no direction is preferred. However, it was surprising for us that beginning with the 6th maximum there must be a dominating positive feedback from the surrounding which causes divergence of $E_x(t, 0, 0, 0)$ (and of all other field components in the origin). We noticed this also for $\hat{p} < 1/16$. Further numerical tests indicated in case of $\hat{p} = \sqrt{\alpha}$ and $t \in [200, 1000]$ an exponential increase of the wave maxima with an average growth factor of about 1.04 per step $t \to t + 1$. The divergence is not surprising after von Neumann Analysis, but we have not expected this delayed kind. Of course it is possible to force a stable development and conservation of energy by simple normalization, e.g. by division of the right sides of (16) by an appropriate function $U(t) \geq 1$, or by application of additional presumptions[7], but we would like to have a good justification for doing that. Remembering 3.1 we guess that more advanced changes are necessary. Although the equations (16) are not exact, they can be suitable for approximative calculation of the initial discrete short term development (see 3.2) because they are directly derived from the Maxwell equations. We have done this without any usage of our knowledge of the fine structure constant, and it is noteworthy that the first maxima of $E_x(t, 0, 0, 0)$ are nearly equal as shown in Fig. 2, if the square of the in (13) defined absolute propagator $\hat{p}$ is equal to the fine structure constant.

5 Conclusions

The (vacuum) Maxwell equations cannot describe physical reality exactly, like all differential equations [11]. Straightforward conversion of the vacuum Maxwell equations into finite-difference equations leads to a system of equations (16) which can serve as first step to a more exact description. It is unambiguous, provides additional combinatorial details and can be suitable

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[7]For example alternating validity of the Maxwell equations. The with this associated alternating algorithms like leapfrog FDTD or ADI (Alternating direct implicit techniques) are common, because they are practical to force stability. But in nature stability is realized by exchange of energy. The with this associated interactions at distributed space time coordinates, however, are difficult to implement. We need more information to simulate e.g. counter-developments which can lead to such interactions.

for approximative calculation of the initial discrete short term development of the electromagnetic fields. The absolute propagator $\hat{p}$ is a new variable which *automatically* arises during formation of the finite differences. In case of $\hat{p} = \sqrt{\alpha}$ a short term wave-like development with nearly equal initial maxima results. If this is not coincidental, the definition of $\hat{p}$ (cf. (13) together with 2.3.1) suggests a new interpretation of the fine structure constant α and (16) can serve as starting point for improvements and further combinatorial studies.

References

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